Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

XXVI.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF HARVARD COLLEGE.

XI. — CHANGE OF ELECTRICAL RESISTANCE IN WIRES BY STRETCHING.

By George S. Pine.

Presented, May 10, 1876.

The electrical resistance of a wire of constant section and material is directly proportional to the length, and inversely proportional to the area of the cross-section. When the wire is stretched, its thickness or cross-section, as well as its length, undergoes a change. This investigation was undertaken to see whether the change in resistance is directly as the length, and inversely as the cross-section of the wire; or whether the copper or the iron, whichever the substance may be, is a better or a worse conductor.

Let l = original length of the wire.
l_1 = length at the end of the experiment.
r = original radius.
r_1 = final radius.
R = original resistance of the wire.
R_1 = final resistance.
v = volume of wire, — supposed constant.
λ = resistance of wire whose length equals its cross-section.

Suppose that λ does not alter. On this supposition, let us find the resultant resistance. We must compare this with the observed resistance to see whether our supposition is right; to see whether λ does or does not alter.

Supposing λ does not vary

$$R = \frac{\lambda l}{\pi r^2} \qquad\qquad R_1 = \frac{\lambda l_1}{\pi r_1^2}$$

$$\frac{R}{R_1} = \frac{\frac{\lambda l}{\pi r^2}}{\frac{\lambda l_1}{\pi r_1^2}} = \frac{l r_1^2}{l_1 r^2} \qquad (1)$$

The volume being unchanged by the stretching

$$\nu = \pi r^2 l \qquad \text{also} \qquad \nu = \pi r_1^2 l_1$$

$$r^2 l = r_1^2 l_1 \qquad \therefore \qquad r_1^2 = r^2 \frac{l}{l_1}$$

Substitute this value in equation (1), —

$$\therefore \qquad \frac{R}{R_1} = \frac{l r^2 l}{l_1 r^2 l_1} = \frac{l^2}{l_1^2}$$

From this we may conclude, that, if the original resistance is to the final resistance in a greater or less ratio than the square of the original length is to the square of the final length, the stress of the particles does alter the specific resistance of the wire. If the ratio of the resistances is greater than the ratio of the squares of the lengths, then the conductivity of the wire is improved.

In the following experiments I used a Thomson's mirror galvanometer with the arrangement of Wheatstone's bridge. I used a wooden bracket attached to a partition of the wall, about 2.5 metres from the floor. I put the wire through a hole in this bracket, and kept it from slipping by driving in a wooden peg. To guard further against slipping, the wire was wound tightly around a screw near the hole. Altogether there was from four to ten centimetres of wire that was not stretched, but whose resistance was taken account of. In the later experiments, this length was taken account of in calculating the resistances. To the lower end of the wire I bound a ring, and on this ring were hung the weights. I connected the ends of the wire with the box of resistance coils by means of thick wires. The resistance of these thick wires was found to be .036 ohms.

Experiment 1. — This experiment was made with copper wire .628 millimetres in diameter. It being the first experiment I made, all the phenomena were not observed. The original and final resistances, and the original and final lengths, I got pretty carefully; but I failed to note the intermediate lengths.

Original length was	1.647 metres.
Final length before breaking	1.857 „
Stretched 21 centimetres.	

	Ohms.
Original resistance with connections	.144
Resistance of connecting wires	.036
Original resistance	.108
Final resistance without connecting wires	.134

$$\frac{l^2}{l_1^2} = \frac{2.713}{3.45} = .786 \qquad \frac{R}{R_1} = \frac{108}{134} = .804 \qquad \frac{R}{R_1} > \frac{l^2}{l_1^2}$$

We should expect R_1 to have been .137. The wire broke under the weight of 16.3 lbs. The weights I used were in lbs., not grammes.

But we have not considered the whole length in considering the change in length; but we have considered the whole length in considering the change in resistance. If we add the same constant to l and l_1, say a, and l_1 is greater than l, then plainly

$$\frac{l^2}{l_1^2} > \frac{(l+a)^2}{(l_1+a)^2}.$$

But more than a may have been added to l_1.

Then, too, if the whole length had been under the stretching process, R_1 might have been greater than it was observed to be. So that the difference of .003 ohms might have been made up, had the whole length been under the stretching process.

In this experiment I measured the final diameter of the wire with the dividing engine, and found that it varied perceptibly in different parts; in one part the mean reading being .593 millimetres, and in another part .56 millimetres. From the deduction given above, it can be seen that we need only consider the lengths and the squares of the lengths. In the following experiments I did not consider the change in diameters.

Experiment 2.— In this experiment I used thin iron wire. 4, 6, 8, and 10 lbs. produced no change in the resistance of the wire, though the length increased slightly. Original resistance was 1.0536 ohms. At first, R was only 1.0584.

10 mm.	$R = 1.068$ ohms.
15 „	$R = 1.0704$ „
20 „	$R = 1.1256$ „
25 „	$R = 1.1304$ „
28 „	$R = 1.1404$ „
30 „	$R = 1.1472$ „

I did not observe the changes in length carefully. At this time, 2 lbs. was the smallest weight I had. On applying fourteen pounds, the wire stretched some; but, when I allowed the whole force to come on, the wire snapped near a place where it was wound. I applied the weight again, and the wire snapped near the middle.

Length of wire before applying 14 lbs. . . . . . . 1.633 metres.
Original length . . . . 1.58 „

$$\frac{l^2}{l_1^2} = \frac{2.496}{2.667} = .927 \qquad \frac{R}{R_1} = \frac{1.0536}{1.1472} = .916$$

Here $\frac{R}{R_1} < \frac{l^2}{l_1^2}$; and it would seem that finally iron has been made a poorer conductor by stretching. As to the intermediate states of the wire, nothing can be inferred. Later I performed another experiment with better results. It was with the same kind of wire. It would seem to show that the conductivity of iron is improved by stretching. I will call it

Experiment 2*a*. — These are the results: —

l	l^2	R
1.696	2.876	1.104
1.716	2.944	1.1064
1.736	3.014	1.1448
1.766	3.119	1.178

$$\frac{l^2}{l_1^2} = .922 \qquad \frac{R}{R_1} = .937 \qquad \frac{R}{R_1} > \frac{l^2}{l_1^2}$$

We should expect R_1 to have been 1.197.

Experiment 3. — In this experiment I used thin copper wire $\frac{11}{40}$ millimetre in diameter. 2 lbs. produced no change in the resistance, though the length increased 2 centimetres. These are the results: —

l	l^2	R
1.476	2.1786	.3816
1.5	2.25	.3816
1.566	2.452	.3912
1.576	2.484	.4032

It broke under weight 4.3 lbs., and I did not get the final results.

$$\frac{l^2}{l_1^2} = .936 \qquad \frac{R}{R_1} = .946$$

Here $\frac{R}{R_1} > \frac{l^2}{l_1^2}$.

It seems that the conductivity of the wire is improved.

I used the same kind of wire in another experiment, which I will call

Experiment 3*a*. — The wire stretched in all 31 centimetres, and broke under the full force of 4.3 lbs. The .3 lbs. is the weight of the iron ring.

	l		l^2	R	
	1.81		3.276	.4776	
	1.905		3.629	.5040	
	1.945		3.783	.5280	
	1.955		3.822	.5520	
	1.985		3.94	.5592	
$l_0 =$	2.055	l_0^2	4.223	.6216	R_0
	2.09		4.368	.624	
	2.12		4.474	.6264	

$$\frac{l^2}{l_1^2} = .729 \qquad \frac{R}{R_1} = .769 \qquad \frac{R}{R_1} > \frac{l^2}{l_1^2}$$

See the curve representing the resistance in this experiment. There is a great and rapid change in the resistance when the wire has been stretched 20 centimetres.

$$\frac{l^2}{l_0^2} = \frac{1.81}{2.055} = .775 \qquad \frac{R}{R_0} = \frac{4776}{6216} \; .769$$

Here at this point $\frac{R}{R_0} < \frac{l^2}{l_0^2}$

Experiment 4. — In this experiment I used thicker copper wire, with the following results : —

l	l^2	R
1.70	2.89	.0636
1.72	2.958	.0636
1.75	3.07	.0648
1.757	3.086	.0648
1.825	3.327	.0684
1.85	3.43	.0768
1.875	3.515	.078

$$\frac{l^2}{l_1^2} = \frac{289}{3.5_15} = .822 \qquad \frac{R}{R_1} = \frac{636}{78} = .828 \qquad \frac{R}{R_1} > \frac{l^2}{l_1^2}$$

I tried the same kind of wire again. It stretched considerably before there was any change in the resistance. The original length was 1.65 metres. The final length was 1.82 metres. 23 lbs. broke the wire before I had time to observe the resistance; but, before applying the last pound, the resistance was .076 ohms. The original resistance was .072.

$$\frac{l^2}{l_1^2} = \frac{2.622}{3.312} = .792 \qquad \frac{R}{R_1} = .947 \qquad \frac{R}{R_1} > \frac{l^2}{l_1^2}$$

Experiment 4*b* was with the same kind of wire, with these results: —

l	l^2	R
1.595	2.544	.0648
1.635		.0648
1.665		.0648
1.715		.0648
1.765	3.116	.0648
1.855	3.44	.0820
1.945	3.783	.090
2.	4.	.1008

$\frac{l^2}{l_1^2} = .636$ $\frac{R}{R_1} = .0.642$ $\frac{R}{R_1} > \frac{l^1}{l_1^2}$

26 lbs. broke the wire.

In all these experiments, it is easy to see that the change in resistance at first is not at all proportional to the increase in length. At the close of some of the experiments, the resistance is almost as much as it ought to be; and perhaps, if there were no error in observation or calculation, the resistance at the breaking-point would be as much as the law would make it.

Experiment 5.—This experiment was made with copper wire not so thick as that used in the previous experiment. I measured the total length, and the changes in length; so that, although not the whole length is under the stretching process, the resistances given correspond more exactly to the lengths given.

l	l^2	R
1.71	2.924	.109
1.725	2.965	.1092
1.755	3.08	.1104
1.815	3.294	.1128
1.84	3.385	.1128
1.875	3.516	.114
1.9	3.61	.114
1.91	3.648	.114
1.95	3.8	.1356
1.99	3.98	.138

$\frac{l^2}{l_1^2} = .738$ $\frac{R}{R_1} = .791$ $\frac{R}{R_1} > \frac{l^2}{l_1^2}$

16 lbs. broke the wire. I performed a second experiment with the same kind of wire, and with essentially the same results.

Experiment 6.—I next took German-silver wire. The curve of observations here will be found to almost coincide with the straight line representing the law. This experiment is quite curious when contrasted with the others.

l	l^2	R
1.53	2.34	4.33
1.535	2.356	4.34
1.545	2.387	4.40
1.57	2.465	4.577
1.595	2.544	4.728
1.615	2.608	4.824
1.64	2.69	4.987
1.665	2.772	5.112
1.69	2.856	5.22
1.735	3.01	5.609

$$\frac{l^2}{l_1^2} = .777 \qquad \frac{R}{R_1} = .77$$

Experiment 6*a*. — German-silver wire again.

Original total length	1.70 metres.
Length to be stretched	1.60 ,,
Original total resistance	4.848
Resistance of connecting wires	.036
Resistance of 1.70 metres	4.812
Subtract $\frac{10}{170}$ of this	.283
Resistance of 1.60 metres	4.529

l	l^2	R
1.60	2.56	4.529
1.61	2.592	4.541
1.64	2.689	4.733
1.67	2.789	4.9226
1.70	2.89	5.1386
1.77	3.133	5.5226
1.815	3.294	5.813
1.85	3.42	6.0506
1.885	2.591	6.2858
1.99	3.96	6.9626

$$\frac{l^2}{l_1^2} = .646 \qquad \frac{R}{R_1} = .650 \qquad \frac{R}{R_1} > \frac{l^2}{l_1^2}$$

By examining the curve, it will be seen that it very nearly follows the law, and that the conductivity of the metal is but little altered.

Experiment 7. — This experiment was made with copper wire No. 18.

Original total length	1.62 metres.
Length to be stretched	1.57 ,,

	Ohm.
Original resistance with connecting wires	.3000
Resistance of connecting wires	.036
Resistance of total length	.264
Resistance of length not stretched	.008
Original resistance of length to be stretched	.256

l	l^2	R
1.57	2.465	.256
1.58		.256
1.595		.256
1.6	2.56	.256
1.615	2.608	.2608
1.64	2.69	.2728
1.66	2.7556	.2776
1.69	2.856	.2776
1.725	2.9756	.2992
1.76	3.0976	.3016

$\frac{l^2}{l_1^2} = \frac{2465}{3097} = .795$ $\frac{R}{R_1} = \frac{256}{3016} = .848$ $\frac{R}{R_1} > \frac{l^2}{l_1^2}$

R_1 should have been .322 if the conductivity of the metal had not been altered.

Experiment 8. — Copper wire No. 22.

Original total length	1.57 metres.
Length to be stretched	1.53 „
Resistance with connecting wires	.3048
Resistance of connecting wires	.036
Resistance of total length	.2688
Resistance of length not stretched	.0068
Resistance of wire to be stretched	.2620

l	l^2	R
1.53	2.341	.262
1.55	2.—	.262
1.59		.262
1.62	2.624	.262
1.655	2.739	.3004
1.68	2.822	.3124

$\frac{l^2}{l_1^2} = .829$ $\frac{R}{R_1} = .841$ $\frac{R}{R_1} > \frac{l^2}{l_1^2}$

The wire broke under weight of 5.2 lbs.

R_1 should have been .3161 if there was no change.

Experiment 8*a*. — With copper wire of same size.

Original length	1.56 metres.
Length to be stretched	1.52 ,,
Resistance of whole length	.2676 ,,
Resistance of length not stretched	.0068 ,,
Resistance of wire to be stretched	.2608 ,,

l	l^2	R
1.52	2.3104	.2608
1.54	2.3716	.2608
1.56	2.434	.2620
1.58	2.496	.2620
1.6	2.56	.2836
1.63	2.65	.2956
1.655	2.706	.298
1.67	2.789	.310
1.69	2.856	.316
1.705	2.907	.3256

$$\frac{l^2}{l_1^2} = .794 \qquad \frac{R}{R_1} = .8$$

Wire broke under 5.2 lbs.

Experiment 9. — *Copper wire, the finest yet.*

Original total length	1.56
Length to be stretched	1.52
Original total resistance	.780
Resistance of connecting wires, &c.	.055

l	l^2	R
1.52	2.3104	.725
1.535		.725
1.55	2.4025	.725
1.565	2.444	.7298
1.59	2.528	.7538
1.61	2.592	.7766
1.64	2.6896	.8018
1.67	2.789	.8318
1.70	2.89	.8618
1.73	2.993	.8918
1.77	3.133	.9314
1.81	3.276	.971
1.855	3.441	1.006

$$\frac{l^2}{l_1^2} = .671 \qquad \frac{R}{R_1} = .7206$$

If there were no change, R_1 should have been 1.08 ohms. The wire broke under pressure of 2.6 lbs.

From the foregoing experiments, I conclude that the conductivity of German-silver wire remains unaltered by stretching; that the conductivity of iron wire is perhaps improved, and that the conductivity of copper wire is improved, that up to a certain point the wire can be stretched without increasing the resistance, or only increasing the resistance very little; that beyond that point the resistance increases very rapidly for a while, and then increases less rapidly. In most cases, after the wire has been stretched to the point where the resistance ceases to increase rapidly, the resistance appears to increase in such a way, that the ratio $\frac{l_1^2}{R_1}$ remains almost constant.

It appears, then, that copper wire can be stretched to some advantage; that, if it is stretched too much, some of the advantage gained is lost again. Experiments *3a* and *8a* are the only instances where the copper wire appears to have lost at any time the full amount of the advantage gained by stretching; but, even in these instances, the wire seems to gain advantage in regard to its conductivity as the stretching goes on.

Thin copper wire being in the process of manufacture, stretched to a great extent, the advantage gained by further stretching is less marked than with thicker wire. It is noteworthy that the wire becomes very brittle before it breaks, and assumes a definite structure like steel wire.